Economics 200B | Winter 2008
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MIDTERM EXAMINATION -- Suggested Answers

1. Using the notation of chapter 12 of Starr's Starr's *General Equilibrium Theory: An introduction*, the production sector of the economy is described this way:

There is a finite family F of firms $j \in F$, each with a technology $Y^j \subset R^N$. An allocation $x^i,\ i \in H$, is attainable if there is $y^j \in Y^j,\ j \in F$, so that $0 \le \sum_{i \in H} x^i \le \sum_{j \in F} y^j + \sum_{i \in H} r^i$ (the inequalities hold co-ordinatewise). A production plan y^j, $j \in F$, is said to be attainable if $0 \le \sum_{j \in F} y^j + \sum_{i \in H} r^i$.

New definition: An attainable production plan y^j is said to be technically efficient if there is no alternative attainable production plan $w^j \in Y^j$, $j \in F$, so that $\sum_{j \in F} w^j \ge \sum_{j \in F} y^j$ with the strict inequality holding in some co-ordinate.

Assume:

Strong Monotonicity (every good is desirable): For each household i $\in$ H, with utility function $u^i(\ )$, $w, x \in X^i$, $w \ge x$ (co-ordinatewise), $w \ne x$, implies $u^i(w) > u^i(x)$.

Note (without proof) that strong monotonicity implies that general equilibrium prices are strictly positive. You may use this observation in the questions below.

Please establish two results:

(i) Under strong monotonicity, technical efficiency of an allocation is a necessary condition for Pareto efficiency.

(ii) Under strong monotonicity, let p, y^j, x^i, $j \in F$, $i \in H$, be a competitive equilibrium. Then y^j is technically efficient (strict positivity of prices may be helpful).

Suggested answer: (i) An attainable consumption allocation x^i, $i \in H$, is said to be Pareto efficient if there is no alternative attainable plan w^i that is Pareto preferable. We will show that if y^j , $j \in F$, is a technically inefficient production *plan and* x^i *a corresponding allocation of output then there is an attainble Pareto preferable allocation --- hence* x^i *is Pareto inefficient.*

We have $\Sigma_i x^i \leq \Sigma_j y^j + r$. *But there is an alternative production plan* v^j *so that* $\Sigma_j y^j \leq \Sigma_j v^j$ *and* $\Sigma_j y^j \neq \Sigma_j v^j$. *Hence there is* w^i *so that* $x^i \leq w^i$, $x^i \neq w^i$, *and* $\Sigma_i w^i \leq \Sigma_j v^j + r$. *But under strong montonicity,* w^i *is Pareto preferable to* x^i. *Hence the technically inefficient* y^j *is incompatible with Pareto efficiency.*

(ii) There are two alternative, equally correct demonstrations:

- *An equilibrium is Pareto efficient. We demonstrated in part i that Pareto efficiency requires technical efficiency. Hence, an equilibrium allocation is technically efficient.*
- *Under strong monotonicity, equilibrium prices are strictly positive. But then profit maximization in the production sector requires technical efficiency in that sector. Let p be equilibrium prices,* y^j *be a technically inefficient production plan and* w^j *a technically superior production plan. Then* $p\bullet\Sigma_j y^j < p\bullet\Sigma_j w^j$. *But then for some firm j' ,* $p\bullet y^{j'} < p\bullet w^{j'}$ *and* $y^{j'}$ *is not an equilibrium production plan.*

2. The following question assumes the pure exchange economy model and the growth of the economy by replication presented in Starr's *General Equilibrium Theory: An introduction*, Chapters 13 and 14. We use assumptions C.IV, C.V, C.VII, C.IX, throughout this question.

Define a new concept, the *balanced core*, as the set of allocations unblocked by *balanced* coalitions.

A blocking coalition S in economy Q-H will be said to *balanced*, if it fulfills two properties:

- it contains the same number of individuals of each type represented in the coalition. For example, a coalition of five households each of types 1, 3, and 7, and zero of all other types fufills this condition; a coalition of five of type 1, four of type 3, one of type 7, and zero of all other types is not balanced.

• equal treatment within the coalition. For any blocking allocation, all households of the same type within the blocking balanced coalition receive the same allocation.

Note that every coalition in 1-H (the original unreplicated economy), is balanced.

We wish to demonstrate that, unlike the core, the balanced core does <u>not</u> converge to the set of competitive equilibria.

Do parts (i), (ii), and (iii):

(i) The balanced core retains two properties of the core: inclusion of the competitive equilibrium (Theorem 13.1), and the equal treatment property (Theorem 14.1). Explain why (this should take no more than a few sentences).

(ii) Prove and explain

Proposition: Let $\{x^{oh} \mid h \in H\}$ be in the core for Q = 1 (the original unreplicated economy). Then $\{x^{oh} \mid h \in H\}$ is in the balanced core of Q-H for all positive integer values of Q.

Hint: Show that if x^{oh} is blocked by a balanced coalition in Q-H, then it is blocked in 1-H (the original unreplicated economy).

(iii) The Proposition in part (ii) means that the balanced core does not converge to the (set of) competitive equilibrium(a). Explain this interpretation.

Suggested answer: (i) The (conventional) core always contains the competitive equilibrium. Any blocking coalition in the balanced core is also blocking in the (conventional) core. The CE is always unblocked in the conventional core so it will be unblocked in the balanced core.

Any unequal treatment allocation is blocked by a balanced coalition consisting of one of each type (proof of Theorem XIV.1). Hence equal treatment is maintained in the balanced core.

(ii) In order to prove the proposition we will demonstrate that any allocation blocked in the balanced core is blocked in the (conventional) core of the original unreplicated economy. Hence any core allocation of the original unreplicated economy is in the balanced core of the replicated economy.

Let x^h , $h \in H$, be an equal treatment allocation. Let y^i, $i \in S \subset H$, be a balanced coalition blocking allocation, with K households of each type. Then we have $K\Sigma_i y^i \leq K\Sigma_i r^i$ (or equivalently, $\Sigma_i y^i \leq \Sigma_i r^i$) and $u^i(y^i) \geq u^i(x^i)$ with $u^i(y^i) > u^i(x^i)$ for some $i \in S$. But this is of course equivalent to blocking in the conventional unreplicated core. Hence any allocation blocked in the balanced core is blocked in the conventional unreplicated core.

(iii) Following (ii) the balanced core of Q-H for Q large is the core for Q=1. The balanced core does not converge.

3. We developed the notion of an Edgeworth Box for the allocation of consumption between two households in Chapter 1 of Starr's *General Equilibrium Theory: An introduction*. We can use the same approach to describe the allocation of inputs to production. Factors of production are analagous to consumption goods in the (consumption) Edgeworth Box, output levels are analagous to household utilities, isoquants are analagous to indifference curves.

Let there be two inputs to production, X and Y, endowed in the amounts $\bar{X}, \bar{Y} > 0$. They are to be allocated between the production of outputs 1 and 2, in the amounts X^1, X^2, Y^1, Y^2 subject to the constraints $X^1 + X^2 = \bar{X}$, $Y^1 + Y^2 = \bar{Y}$. They produce outputs 1 and 2 according to the production functions

$$Q^1 = F(X^1,Y^1) = [X^1Y^1]^{1/2}, \qquad Q^2 = G(X^2,Y^2) = [X^2Y^2]^{1/3}.$$

(The superscripts $^1/_2$ and $^1/_3$ are exponents; the superscripts 1 and 2 indicate the product). The allocation of inputs to production is *technically efficient* if there is no reallocation of inputs that would allow an increase in output of 1 or 2 without a reduction in output of the other.

(a) Consider an Edgeworth Box with sides of length $\bar{X}$ and $\bar{Y}$. Let opposite corners of the box depict two allocations, one with all resources going to produce 1 and the other with all resources going to produce 2. Describe the locus of tangencies of the isoquants. This locus represents the technically efficient allocation of resources to production. Explain why.

(b) Let the factors sell for p_x and p_y, with each firm choosing its input mix to minimize the cost of inputs for each level of output. Show that a factor market equilibrium will lie on the locus of tangencies.

(c) The production possibility set (bounded by the production frontier) in output (good 1 - good 2) space is defined as

$PPS = \{(Q^1,Q^2) \mid Q^1 \leq F(X^1,Y^1) = [X^1Y^1]^{1/2},\ Q^2 \leq G(X^2,Y^2) = [X^2Y^2]^{1/3};\ X^1 + X^2 = \bar{X},\ Y^1 + Y^2 = \bar{Y}\}$

Describe this set. What is the relationship of the production frontier to the locus of isoquant tangencies in the Edgeworth Box?

Suggested Answer:

(a) Consider the problem

Max $Q^1 = F(X^1,Y^1) = [X^1Y^1]^{1/2}$ subject to $Q^2 = G(X^2,Y^2) = [X^2Y^2]^{1/3} = K^*$ (a real number)

and subject to $X^1 + X^2 = \bar{X}$, $Y^1 + Y^2 = \bar{Y}$

Characterizing the class of solutions to this problem is characterizing the set of technically efficient allocations. To solve this problem form the Lagrangian

$L \equiv F(X^1, Y^1) + \lambda[G(\bar{X} - X^1, \bar{Y} - Y^1) - K^*]$. Conditions for an extremum then are

$$\frac{\partial L}{\partial X^1} = \frac{\partial F}{\partial X^1} - \lambda \frac{\partial G}{\partial X^2} = 0$$

$$\frac{\partial L}{\partial Y^1} = \frac{\partial F}{\partial Y^1} - \lambda \frac{\partial G}{\partial Y^2} = 0$$

$$\frac{\partial L}{\partial \lambda} = G(X^2, Y^2) - K^* = 0.$$

This gives us then the condition

$$\frac{\frac{\partial F}{\partial X^1}}{\frac{\partial F}{\partial Y^1}} = \frac{\frac{\partial G}{\partial X^2}}{\frac{\partial G}{\partial Y^2}}$$ or equivalently

$$\frac{\partial Y^1}{\partial X^1}\Big|_{F=\text{constant}} = \frac{\partial Y^2}{\partial X^2}\Big|_{G=\text{constant}}$$

What this expression says is that technically efficient allocations are characterized by the slope of the isoquants for F equaling the slope of the isoquants for G. That is, technically efficient allocations are the locus of tangencies of the isoquants of F and G.

(b) Cost minimization for a given output level will be characterized by

$$\frac{p_x}{p_y} = \frac{\frac{\partial F}{\partial X^1}}{\frac{\partial F}{\partial Y^1}} = \frac{\frac{\partial G}{\partial X^2}}{\frac{\partial G}{\partial Y^2}}$$

So cost minimization leads to a technically efficient allocation. The requirement of factor market clearing in equilibrium assures us that allocation then is on the locus of tangencies.

(c) The outer frontier of the set PPS is the combination of outputs Q^1, Q^2 consistent with resource endowment and technical efficiency. Each point on the locus of tangencies in the input Edgeworth Box corresponds to an output combination on this frontier.

4. The conventional economic model of the firm in intermediate economics textbooks has a U-shaped cost curve reflecting a small nonconvexity in the technology at low levels of output. This is contrary to the conventional general equilibrium model that typically requires convexity of production technology. Applying the general equilibrium model (e.g. of Starr's *General Equilibrium Theory*, chaps 4-12) to an economy with firms having U-shaped cost curves, can you conclude:

a. there is <u>no</u> general equilibrium?

b. even if a general equilibrium exists, the allocation of resources in equilibrium may not be Pareto efficient?

c. there <u>may not</u> exist a general equilibrium, but it is possible that one exists?

d. if a general equilibrium exists, then the cost curves were not really U-shaped; there is no nonconvexity in the technology?

Explain your answer in each part.

Suggested Answer:

This question does not refer to approximate equilibria of a large economy.

a. No. We cannot conclude that there is no general equilibrium. The conditions in the existence of general equilibrium theorems are sufficient, not necessary.

b. No. We cannot conclude that a general equilibrium may occur with inefficient resource allocation. The First Fundamental Theorem of Welfare Economics applies even without assuming convexity, so a general competitive equilibrium allocation will be Pareto efficient.

c. Yes. In this setting there may not exist a competitive equilibrium. But existence of equilibrium is certainly possible, as in part a.

d. No. This inference is not valid. See part a.

5. In discussing the relationship of saving to consumption in a monetary economy, Keynes writes

"An act of individual saving means --- so to speak --- a decision not to have dinner to-day. But it does *not* necessitate a decision to have dinner or to buy a pair of boots a week hence or a year hence or to consume any specified thing at any specified date. Thus it depresses the business of preparing to-day's dinner without stimulating the business of making ready for some future act of consumption...If saving consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect might indeed be different."

--- J. M. Keynes, *The General Theory...*, chap. 16.

Keynes is arguing that a saving decision implies an intention to postpone consumption but that intention does not show up as a demand for future consumption in the economy. Can the difficulty Keynes notes ("depresses the business of preparing to-day's consumption without stimulating ... some future act of consumption") occur in an Arrow-Debreu economy with a full set of futures markets ? Explain.

Suggested Answer: In the Arrow-Debreu economy, the household faces a single budget constraint for the value of all current and future consumption. In this economy, a household reducing expenditure on goods deliverable in the present will find that it has additional purchasing power, that it does not wish to waste. It will then increase spending on some other consumption item, goods deliverable in the future (we omit the possibility of rearranging past consumption). In this sense, deferring current consumption ('saving' to Keynes) does mean 'not merely ... abstaining from present consumption but ... placing simultaneously a specific order for future consumption.' Hence, Keynes's difficulty cannot arise in the Arrow-Debreu economy.